Energies and temperatures are in eV; all other units are cgs except where noted. Z is the charge state ($Z = 0$ refers to a neutral atom); the subscript e labels electrons. N refers to number density, n to principal quantum number. Asterisk superscripts on level population densities denote local thermodynamic equilibrium (LTE) values. Thus $N_n{}^*$ is the LTE number density of atoms (or ions) in level n.

Characteristic atomic collision cross section:

$$\pi {a_0}^2 = 8.80 \times 10^{-17}\,\mathrm{cm}^2. \tag{1}$$

Binding energy of outer electron in level labelled by quantum numbers n, l:

$$E_\infty^Z(n,l) = -\frac{Z^2 E_\infty^H}{(n-\Delta_l)^2}, \tag{2}$$

where $E_\infty^H = 13.6\,\mathrm{eV}$ is the hydrogen ionization energy and $\Delta_l = 0.75 l^{-5}$, $l \gtrsim 5$, is the quantum defect.

Excitation and Decay

Cross section (Bethe approximation) for electron excitation by dipole allowed transition $m \to n$ (Refs. 32, 33):

$$\sigma_{mn} = 2.36 \times 10^{-13} \frac{f_{nm}\, g(n,m)}{\epsilon \Delta E_{nm}}\,\mathrm{cm}^2, \tag{3}$$

where f_{nm} is the oscillator strength, $g(n,m)$ is the Gaunt factor, ϵ is the incident electron energy, and $\Delta E_{nm} = E_n - E_m$.

Electron excitation rate averaged over Maxwellian velocity distribution, $X_{mn} = N_e \langle \sigma_{mn} v \rangle$ (Refs. 34, 35):

$$X_{mn} = 1.6 \times 10^{-5} \frac{f_{nm} \langle g(n,m) \rangle N_e}{\Delta E_{nm} T_e^{1/2}} \exp\left(-\frac{\Delta E_{nm}}{T_e}\right)\,\mathrm{sec}^{-1}, \tag{4}$$

where $\langle g(n,m) \rangle$ denotes the thermal averaged Gaunt factor (generally ~ 1 for atoms, ~ 0.2 for ions).

Rate for electron collisional deexcitation:

$$Y_{nm} = (N_m{*}/N_n{*})X_{mn}. \tag{5}$$

Here $N_m{*}/N_n{*} = (g_m/g_n)\exp(\Delta E_{nm}/T_e)$ is the Boltzmann relation for level population densities, where g_n is the statistical weight of level n.

Rate for spontaneous decay $n \to m$ (Einstein A coefficient)[34]

$$A_{nm} = 4.3 \times 10^7 (g_m/g_n) f_{mn} (\Delta E_{nm})^2 \sec^{-1}. \tag{6}$$

Intensity emitted per unit volume from the transition $n \to m$ in an optically thin plasma:

$$I_{nm} = 1.6 \times 10^{-19} A_{nm} N_n \Delta E_{nm} \text{ watt/cm}^3. \tag{7}$$

Condition for steady state in a corona model:

$$N_0 N_e \langle \sigma_{0n} v \rangle = N_n A_{n0}, \tag{8}$$

where the ground state is labelled by a zero subscript.

Hence for a transition $n \to m$ in ions, where $\langle g(n,0) \rangle \approx 0.2$,

$$I_{nm} = 5.1 \times 10^{-25} \frac{f_{nm} g_0 N_e N_0}{g_m T_e^{1/2}} \left(\frac{\Delta E_{nm}}{\Delta E_{n0}} \right)^3 \exp\left(-\frac{\Delta E_{n0}}{T_e} \right) \frac{\text{watt}}{\text{cm}^3}. \tag{9}$$

Ionization and Recombination

In a general time-dependent situation the number density of the charge state Z satisfies

$$\frac{dN(Z)}{dt} = N_e \Big[-S(Z)N(Z) - \alpha(Z)N(Z) + S(Z-1)N(Z-1) + \alpha(Z+1)N(Z+1) \Big]. \tag{10}$$

Here $S(Z)$ is the ionization rate. The recombination rate $\alpha(Z)$ has the form $\alpha(Z) = \alpha_r(Z) + N_e \alpha_3(Z)$, where α_r and α_3 are the radiative and three-body recombination rates, respectively.

Classical ionization cross-section[36] for any atomic shell j

$$\sigma_i = 6 \times 10^{-14} b_j g_j(x)/{U_j}^2 \text{ cm}^2. \tag{11}$$

Here b_j is the number of shell electrons; U_j is the binding energy of the ejected electron; $x = \epsilon/U_j$, where ϵ is the incident electron energy; and g is a universal function with a minimum value $g_{\min} \approx 0.2$ at $x \approx 4$.

Ionization from ion ground state, averaged over Maxwellian electron distribution, for $0.02 \lesssim T_e/E_\infty^Z \lesssim 100$ (Ref. 35):

$$S(Z) = 10^{-5} \frac{(T_e/E_\infty^Z)^{1/2}}{(E_\infty^Z)^{3/2}(6.0 + T_e/E_\infty^Z)} \exp\left(-\frac{E_\infty^Z}{T_e}\right) \text{ cm}^3/\text{sec}, \tag{12}$$

where E_∞^Z is the ionization energy.

Electron-ion radiative recombination rate $(e + N(Z) \to N(Z-1) + h\nu)$ for $T_e/Z^2 \lesssim 400\,\text{eV}$ (Ref. 37):

$$\alpha_r(Z) = 5.2 \times 10^{-14} Z \left(\frac{E_\infty^Z}{T_e}\right)^{1/2} \left[0.43 + \frac{1}{2}\ln(E_\infty^Z/T_e) + 0.469(E_\infty^Z/T_e)^{-1/3}\right] \text{ cm}^3/\text{sec}. \tag{13}$$

For $1\,\text{eV} < T_e/Z^2 < 15\,\text{eV}$, this becomes approximately[35]

$$\alpha_r(Z) = 2.7 \times 10^{-13} Z^2 {T_e}^{-1/2} \text{ cm}^3/\text{sec}. \tag{14}$$

Collisional (three-body) recombination rate for singly ionized plasma:[38]

$$\alpha_3 = 8.75 \times 10^{-27} {T_e}^{-4.5} \text{ cm}^6/\text{sec}. \tag{15}$$

Photoionization cross section for ions in level n, l (short-wavelength limit):

$$\sigma_{\text{ph}}(n,l) = 1.64 \times 10^{-16} Z^5/n^3 K^{7+2l} \text{ cm}^2, \tag{16}$$

where K is the wavenumber in Rydbergs (1 Rydberg $= 1.0974 \times 10^5$ cm^{-1}).

Ionization Equilibrium Models

Saha equilibrium:[39]

$$\frac{N_e N_1{}^*(Z)}{N_n{}^*(Z-1)} = 6.0 \times 10^{21} \frac{g_1^Z T_e{}^{3/2}}{g_n^{Z-1}} \exp\left(-\frac{E_\infty^Z(n,l)}{T_e}\right) \text{ cm}^{-3}, \tag{17}$$

where g_n^Z is the statistical weight for level n of charge state Z and $E_\infty^Z(n,l)$ is the ionization energy of the neutral atom initially in level (n,l), given by Eq. (2).

In a steady state at high electron density,

$$\frac{N_e N^*(Z)}{N^*(Z-1)} = \frac{S(Z-1)}{\alpha_3}, \tag{18}$$

a function only of T.

Conditions for LTE:[39]

(a) Collisional and radiative excitation rates for a level n must satisfy

$$Y_{nm} \gtrsim 10 A_{nm}. \tag{19}$$

(b) Electron density must satisfy

$$N_e \gtrsim 7 \times 10^{18} Z^7 n^{-17/2} (T/E_\infty^Z)^{1/2} \text{cm}^{-3}. \tag{20}$$

Steady state condition in corona model:

$$\frac{N(Z-1)}{N(Z)} = \frac{\alpha_r}{S(Z-1)}. \tag{21}$$

Corona model is applicable if[40]

$$10^{12} t_I{}^{-1} < N_e < 10^{16} T_e{}^{7/2} \text{ cm}^{-3}, \tag{22}$$

where t_I is the ionization time.

Radiation

N. B. Energies and temperatures are in eV; all other quantities are in cgs units except where noted. Z is the charge state ($Z = 0$ refers to a neutral atom); the subscript e labels electrons. N is number density.

Average radiative decay rate of a state with principal quantum number n is

$$A_n = \sum_{m<n} A_{nm} = 1.6 \times 10^{10} Z^4 n^{-9/2}\,\mathrm{sec.} \tag{23}$$

Natural linewidth (ΔE in eV):

$$\Delta E\,\Delta t = h = 4.14 \times 10^{-15}\,\mathrm{eV\,sec}, \tag{24}$$

where Δt is the lifetime of the line.

Doppler width:

$$\Delta\lambda/\lambda = 7.7 \times 10^{-5}(T/\mu)^{1/2}, \tag{25}$$

where μ is the mass of the emitting atom or ion scaled by the proton mass.

Optical depth for a Doppler-broadened line:[39]

$$\tau = 3.52\times10^{-13} f_{nm}\lambda(Mc^2/kT)^{1/2}NL = 5.4\times10^{-9} f_{nm}\lambda(\mu/T)^{1/2}NL, \tag{26}$$

where f_{nm} is the absorption oscillator strength, λ is the wavelength, and L is the physical depth of the plasma; M, N, and T are the mass, number density, and temperature of the absorber; μ is M divided by the proton mass. Optically thin means $\tau < 1$.

Resonance absorption cross section at center of line:

$$\sigma_{\lambda=\lambda_c} = 5.6 \times 10^{-13}\lambda^2/\Delta\lambda\,\mathrm{cm}^2. \tag{27}$$

Wien displacement law (wavelength of maximum black-body emission):

$$\lambda_{\max} = 2.50 \times 10^{-5} T^{-1}\,\mathrm{cm}. \tag{28}$$

Radiation from the surface of a black body at temperature T:

$$W = 1.03 \times 10^5 T^4\,\mathrm{watt/cm}^2. \tag{29}$$

Bremsstrahlung from hydrogen-like plasma:[26]

$$P_{\mathrm{Br}} = 1.69 \times 10^{-32} N_e {T_e}^{1/2} \sum \left[Z^2 N(Z) \right] \text{ watt/cm}^3, \tag{30}$$

where the sum is over all ionization states Z.

Bremsstrahlung optical depth:[41]

$$\tau = 5.0 \times 10^{-38} N_e N_i Z^2 \overline{g} L T^{-7/2}, \tag{31}$$

where $\overline{g} \approx 1.2$ is an average Gaunt factor and L is the physical path length.

Inverse bremsstrahlung absorption coefficient[42] for radiation of angular frequency ω:

$$\kappa = 3.1 \times 10^{-7} Z {n_e}^2 \ln \Lambda \, T^{-3/2} \omega^{-2} (1 - \omega_p^2/\omega^2)^{-1/2} \text{ cm}^{-1}; \tag{32}$$

here Λ is the electron thermal velocity divided by V, where V is the larger of ω and ω_p multiplied by the larger of Ze^2/kT and $\hbar/(mkT)^{1/2}$.

Recombination (free-bound) radiation:

$$P_r = 1.69 \times 10^{-32} N_e {T_e}^{1/2} \sum \left[Z^2 N(Z) \left(\frac{E_\infty^{Z-1}}{T_e} \right) \right] \text{ watt/cm}^3. \tag{33}$$

Cyclotron radiation[26] in magnetic field $\mathbf{B}$:

$$P_c = 6.21 \times 10^{-28} B^2 N_e T_e \text{ watt/cm}^3. \tag{34}$$

For $N_e k T_e = N_i k T_i = B^2/16\pi$ ($\beta = 1$, isothermal plasma),[26]

$$P_c = 5.00 \times 10^{-38} N_e^2 T_e^2 \text{ watt/cm}^3. \tag{35}$$

Cyclotron radiation energy loss e-folding time for a single electron:[41]

$$t_c \approx \frac{9.0 \times 10^8 B^{-2}}{2.5 + \gamma} \text{ sec}, \tag{36}$$

where γ is the kinetic plus rest energy divided by the rest energy mc^2.

Number of cyclotron harmonics[41] trapped in a medium of finite depth L:

$$m_{\mathrm{tr}} = (57 \beta B L)^{1/6}, \tag{37}$$

where $\beta = 8\pi N k T/B^2$.

Line radiation is given by summing Eq. (9) over all species in the plasma.